Math 10

Lesson 1-1 Factors, primes, composites and multiples

I. Factors and common factors

Problem:

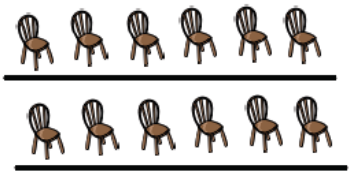
If you were asked to arrange 12 chairs in a classroom, how many different arrangements of the chairs would be possible? (The chairs must form one group and there cannot be any remaining (remainder) chairs.)

Solutions:

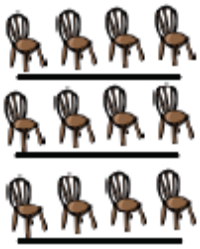
One solution would be to draw all the possible arrangements of the 12 chairs.

1 row of 12 chairs

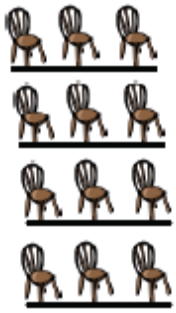
2 rows of 6 chairs



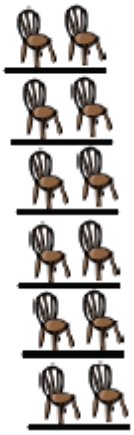
3 rows of 4 chairs



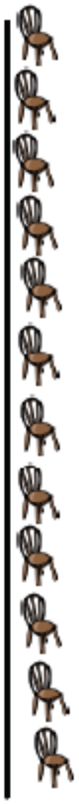
4 rows of 3 chairs



6 rows of 2 chairs



12 rows of 1 chairs

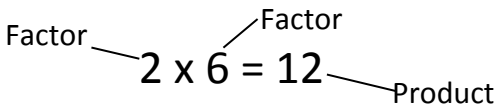


Drawing the chairs is a perfectly acceptable way of solving the chair arranging problem, especially if you love drawing chairs and you have a lot of time on your hands. But what if the number of chairs is now 260, how many arrangements are possible? Drawing 260 chairs into different arrangements is far more tedious. Perhaps there is a better way …

Factors

Returning to the 12 chair problem, perhaps we can use the idea of **factors**. Recall what a factor is:

A factor is any number that is multiplied by another number to get a product
For example, 2 and 6 are factors of 12.



OR
Another way to say it is: a factor is any number that divides into another number exactly.
For example, since 4 divides into 12 exactly, 4 is a factor of 12.

Using the numbers from 1 to 12 we find that:

1 x 12 = 12
2 x 6 = 12
3 x 4 = 12
4 x 3 = 12
6 x 2 = 12
12 x 1 = 12

Note that these factors are another way to represent the ways that the chairs can be rearranged.

Therefore, the **factors** of 12 are 1, 2, 3, 4, 6, 12

Question 1
Find the factors of 36.

Question 2
How many different ways are there to arrange 260 chairs?

Question 3
Find the factors of 17.

Common Factors

Quite often we are interested in finding factors that different numbers have in common. For example the common factors for 12 and 16 are 1, 2 and 4.

Example 1 Determine the common factors of 12, 15 and 21.

There are many ways to solve this problem. One approach is to determine the factors of each number and then compare:

1 x 12 = 12	**1** x 15 = 15	**1** x 21 = 21
2 x 6 = 12	**3** x 5 = 15	**3** x 7 = 21
3 x 4 = 12		

∴ the common factors for 12, 15 and 21 are 1 and 3

II. Primes, composites and multiples

Prime numbers

Note for Question 3 that the only factors for 17 are 1 and 17. **A number with only two factors is a <u>prime number</u>. The two factors of a prime number are the number and one**. 17 is a prime number.

Question 4

Identify all of the prime numbers between 2 and 20.

2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Composite numbers

A number with three or more factors is a <u>composite number</u>. For example, 8 is a composite number since it has more than two factors

$1 \cdot 8 = 8$

$2 \cdot 4 = 8$

The factors of 8 are 1, 2, 4 and 8. 8 is a composite number.

Question 5

Is 18 a prime number or a composite number?

Question 6

After 10, what are the next three composite numbers that are odd.

Question 7

A wonderful tradition that has developed over the years is for Dr. Licht's students to contribute to "Dr. Licht's summer golf fund" a tax deductible, not-for-profit fund that contributes to the well-being of Dr. Licht. If your class were to donate a looney every week and the semester has 87 teaching days, how many looneys will be in the fund at the end of the semester?

Multiples

A multiple of a number is the product of a given number and an integer value other than 0.
For example, the multiples of 5 are 5, 10, 15, 20, …

Question 8

Find the first six multiples of 13

Example 2 Find the first three common multiples of 12 and 18.

The multiples of 12 and 18 are:

12, 24, **36**, 48, 60, **72**, 84, 96, **108**, …
18, **36**, 54, **72**, 90, **108**, …

∴ the first three common multiples of 12 and 18 are **36**, **72**, and **108**

III. Assignment

1. List the first 6 multiples of each number.

a) 5 **b)** 11 **c)** 18 **d)** 25

2. Determine the first 3 common multiples of each pair of numbers.

a) 2 and 5 **b)** 3 and 9 **c)** 7 and 3 **d)** 8 and 10

3. Determine the factors of each number. List the factors that are prime numbers.

a) 15 **b)** 20 **c)** 24

d) 45 **e)** 60 **f)** 100

4. Determine the common factors of each pair of numbers.

a) 16 and 24 **b)** 15 and 45 **c)** 18 and 42 **d)** 20 and 30

5. Which of the numbers from 2 to 130 are prime numbers? (You may want to use the Sieve of Eratosthenes on the next page. Either ask your teacher or Google it to see how the sieve works.)

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130